United States
Department of
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Forest Service

**Intermountain
Forest and Range
Experiment Station**

Research Note
INT-310

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Spot Fire Distance Equations for Pocket Calculators

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ABSTRACT

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This note presents equations for calculating maximum spot fire distance from firebrand sources in the Intermountain West based on prevailing windspeed, vegetation cover, and terrain in the area. The equations include the capability to predict spotting distance from a torching tree(s) or from a continuous flame source such as slash piles or jackpots of heavy fuels. The equations can be used on a programmable pocket calculator. Potential uses are seen in fire management planning and real-time fire behavior predictions. For copies of a program for the Texas Instruments TI-59, send seven blank TI-59 magnetic cards to the author.

KEYWORDS: spot fire, spotting, firebrands, fire management

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[1]The author is a mathematician stationed at the Intermountain Station's Northern Forest Fire Laboratory, Missoula, Mont.

The ability to predict spot fire distance from a firebrand source gives fire management personnel another work tool. This research note presents equations for predicting maximum spotting distance from sources such as torching trees and burning piles of debris in the Intermountain West. Tree species considered have been confined to those common to the Intermountain West where data and personal experience of researchers corroborate the relationships used. To provide a simple mechanized process of calculation, these equations can be programed on a pocket calculator. Here the calculation process is implemented on the programmable Texas Instruments TI-59.

The equations are based on the model by Albini (1979), who employed a nomogram solution. The nomogram solution was simplified and used as the basis of the Fire Behavior Officers' (FBO) field procedure.[2] The original model has been extended (Albini 1981) to apply when transport of the firebrand is over nonforested terrain, or when the flame source is continuous (such as slash piles, jackpots of heavy fuels) as opposed to the brief, transitory flame from a torching tree. The spotting model is applicable under conditions of intermediate fire severity in which spotting distances up to a mile or two might be expected, and intensities would be sufficiently high to occasionally involve overstory fuels.

The equations presented here are those for the FBO field procedure with these additional features:

1. Multiplication factors to account for simultaneous burning of several identical trees that produces a single merged flame.

2. Correction of flat-terrain spotting distance for use in mountainous terrain.

3. Prediction of spotting distance from a continuous flame source.

4. Allowance for transport over short grass, bare ground, or water as appropriate.

The assumption was made in the FBO field procedure that wind at treetop level is two-thirds of the wind 20 ft (6 m) above the canopy. That assumption is retained here.

In his model, Albini had six submodels, each based on a host of assumptions. Not all those assumptions need to be stated here. It is sufficient to recognize that Albini described ideal conditions to maximize firebrand transport distance--firebrands sufficiently small to be carried some distance, yet large enough to still be viable when coming to rest, wind blowing steadily in one direction, and so forth. Conditions as they exist in the field seldom conform to this ideal. For example, winds in forested mountains vary greatly both in speed and direction. However, any variation from the ideal assumed in the model would serve only to decrease the spot fire distance. The user should expect actual spotting distance to fall short of the prediction when deviation is observed.

Albini points out factors not considered in his approach:

1. *Probability of tree(s) torching out.* Instead, the model describes what would happen *if* trees torch out.

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[2]National Wildfire Coordinating Group. Fire behavior officers' (FBO) field reference. USDA For. Serv. National Adv. Resource Technology Center, Marana Air Park, AZ, looseleaf.

2. *Availability of optimum firebrand material*. There may or may not be a firebrand particle capable of being carried the maximum distance.

3. *Probability of spot fire ignition*. Not addressed is whether the firebrand material lands in an area with easily ignited fuels, and whether enough spark or ember remains to cause ignition.

4. *Number of spot fires*. More information about the first three items is needed before this question can be considered.

The purpose of this note is to document the equations used in mechanizing the calculation of spotting distance. The remainder of the text presents the equations used, the calculator program, and operating procedures. Development of the equations is given in the appendix. The TI-59 program uses U.S. units of measure, so only U.S. units appear in this text. However, in the summary of equations (next section), metric units are added for those who may be interested.

SUMMARY OF EQUATIONS

Symbol	U.S. units	Metric units	Description
d	inch	cm	Diameter at breast height (d.b.h.) of tree(s) torching out
h	ft	m	Height of burning tree(s)
$\bar{h}$	ft	m	Mean vegetation cover height downwind of source
$\bar{h}_c$	ft	m	Minimum value of $\bar{h}$ used to calculate spotting distance using the logarithmic windspeed variation with height (Albini 1981)
$\bar{h}^*$	ft	m	The greater of $\bar{h}$ and $\bar{h}_c$
U	mi/h	km/h	Windspeed 20 feet (6 m) above vegetation
n	none	none	Number of trees burning simultaneously to produce a single merged flame and buoyant plume structure
h_F	ft	m	Adjusted steady flame height (perpendicular measurement from base of flame to tip of flame)
d_F	none	none	Adjusted steady flame duration
H_f	ft	m	Continuous flame height for pile burning
$z(0)$	ft	m	Initial firebrand height above ground
F	mi	km	Flat-terrain spotting distance
D	mi	km	Ridge-to-valley horizontal distance (map)
H	1000's ft	multiples of 300 m	Ridge-to-valley elevational difference
S	mi	km	Mountainous-terrain spotting distance (map)
M	none	none	Code number for location of firebrand source 0=midslope, windward side 1=valley bottom 2=midslope, leeward side 3=ridgetop

Equations Using U.S. Units

$$h_F = \begin{cases} 16.5d^{0.515}n^{0.4}, & \text{grand fir, balsam fir} \\ 15.7d^{0.451}n^{0.4}, & \text{Engelmann spruce, subalpine fir, Douglas-fir, western hemlock} \\ 12.9d^{0.453}n^{0.4}, & \text{ponderosa pine, lodgepole pine, white pine} \end{cases}$$

$$d_F = \begin{cases} 12.6d^{-0.256}n^{-0.2}, & \text{ponderosa pine, lodgepole pine, Engelmann spruce} \\ 10.7d^{-0.278}n^{-0.2}, & \text{subalpine fir, Douglas-fir, balsam fir, grand fir, white pine} \\ 6.3d^{-0.249}n^{-0.2}, & \text{western hemlock} \end{cases}$$

$$z(0) = \begin{cases} 4.24d_F^{0.332}(h_F) + h/2, & h/h_F \geq 1 \\ 3.64d_F^{0.391}(h_F) + h/2, & 0.5 \leq h/h_F < 1 \\ 2.78d_F^{0.418}(h_F) + h/2, & h/h_F < 0.5,\ d_F < 3.5 \\ 4.70(h_F) + h/2 & h/h_F < 0.5,\ d_F \geq 3.5 \\ 12.2H_F, & \text{pile burning option} \end{cases}$$

$$\overline{h}_c = 2.2\ z(0)^{0.337} - 4.0$$

$$\overline{h}^* = \max(\overline{h},\overline{h}_c)$$

$$F = 7.18 \times 10^{-4} U\overline{h}^{*1/2}\left\{0.362 + \left(\frac{z(0)}{\overline{h}^*}\right)^{1/2} \frac{1}{2} \ln\left(\frac{z(0)}{\overline{h}^*}\right)\right\}$$

$S = D \cdot X_6$, where X_6 is from the iteration:

$$X_o = A$$

$$X_{n+1} = A - B\left(\cos(\pi X_n - M\pi/2) - \cos(M\pi/2)\right)$$

$$A = F/D$$

$$B = H/(10\pi)$$

Equations Using Metric Units

$$h_F = \begin{cases} 3.11 d^{0.515} n^{0.4}, & \text{grand fir, balsam fir} \\ 3.14 d^{0.451} n^{0.4}, & \text{Engelmann spruce, subalpine fir, Douglas-fir, western hemlock} \\ 2.58 d^{0.453} n^{0.4}, & \text{ponderosa pine, lodgepole pine, white pine} \end{cases}$$

$$d_F = \begin{cases} 16.0 d^{-0.256} n^{-0.2}, & \text{ponderosa pine, lodgepole pine, Engelmann spruce} \\ 13.9 d^{-0.278} n^{-0.2}, & \text{subalpine fir, Douglas-fir, balsam fir, grand fir, white pine} \\ 7.95 d^{-0.249} n^{-0.2}, & \text{western hemlock} \end{cases}$$

$$z(0) = \begin{cases} 4.24 d_F^{0.332} (h_F) + h/2, & h/h_F \geq 1 \\ 3.64 d_F^{0.391} (h_F) + h/2, & 0.5 \leq h/h_F < 1 \\ 2.78 d_F^{0.418} (h_F) + h/2, & h/h_F < 0.5,\ d_F < 3.5 \\ 4.70 (h_F) + h/2, & h/h_F < 0.5,\ d_F \geq 3.5 \\ 12.2 H_F, & \text{pile burning option} \end{cases}$$

$$\bar{h}_c = z(0)^{0.337} - 1.22$$

$$\bar{h}^* = \max(\bar{h}, \bar{h}_c)$$

$$F = 1.30 \times 10^{-3} U \bar{h}^{*1/2} \left\{ 0.362 + \left(\frac{z(0)}{\bar{h}^*}\right)^{1/2} \frac{1}{2} \ln\left(\frac{z(0)}{\bar{h}^*}\right) \right\}$$

$S = D \cdot X_6$, where X_6 is from the iteration:

$$X_o = A$$

$$X_{n+1} = A - B\left(\cos(\pi X_n - M\pi/2) - \cos(M\pi/2)\right)$$

$$A = F/D$$

$$B = H/(10\pi)$$

The program is recorded on one card (magnetic strip) and is accompanied by six data cards for various tree species. The TI-59 program can be used with data cards for species other than those shown in the equation summary. A prerequisite is that, for any species, the curves for flame height and flame duration can be represented as power curves of the form $y = ax^b$. (See the appendix for directions on how to make data cards for other tree species.

Copies of the program may be obtained by sending seven blank magnetic cards for the TI-59 to the author at the Northern Forest Fire Laboratory.

The program has two options:

1. The firebrand source is a burning tree(s) that is torching out--torching tree option.

2. The source is a burning pile of debris (or fuel of similar nature) where the flame is "continuous"--pile burning option.

Input requirements, listed below, differ for the options. (Guidelines for selecting input values are given in the section on operating procedure.)

Torching tree option	Pile burning option
Species data	Mean cover height
Torching tree d.b.h.	20-foot windpseed
Torching tree height	Observed flame height
Mean cover height	Ridge/valley elevation difference
20-foot windspeed	Ridge/valley horizontal distance
Number of trees (burning at once)	Spotting source location code
Ridge/valley elevation difference	
Ridge/valley horizontal distance	
Spotting source location code	

There are no checks made on the validity of input data. The operator must be careful to select reasonable values and to enter those data without error.

OPERATING PROCEDURE

The spotting distance program may be run with any solid state module in the calculator (such as NFDRS/Fire Behavior module, library module, statistics module) or with no module in the calculator.

I. Preliminaries

1. Turn on the calculator. If it is already on, turn it off momentarily to clear program and data registers.

2. Press $\boxed{1}$. Feed side 1 of the program card into the lower slot on the right side of the calculator. The motor will start and stop automatically. If the display flashes, press $\boxed{\text{CLR}}$ and repeat step 2.

3. Press [2]. Feed side 2 of the program card into the slot. If the display flashes, press [CLR] and repeat step 3.

II. Initialization and input

1. Input species data (omit for pile burning option): Press [SBR] [STO] and feed either side of the data card for the desired species into the TI-59 (the data are recorded on both sides of the card). A 4. should appear in the display; if the display flashes, press [CLR] and try again. These data remain in the TI-59 until replaced with data for another species by simply repeating this step with a different species card. Species data need not be cleared to run the pile burning option.

2. Press [SBR] [CLR]. This should be done before each run to clear subroutine return registers and to clear flags that signal operating options. Although it is only necessary following a run of the pile burning option or when a run was manually stopped before its normal termination, it is better to make it a part of routine procedure to prevent problems. With the exception of continuous flame height for pile burning, it does not remove inputs. The continuous flame height is reset to 0 by this step.

3. Record the required inputs on the worksheet (fig. 1). Except for continuous flame height, the presence of additional inputs that are not required does not affect calculations. Enter the required items in any order as follows:

--Enter diameter at breast height (d.b.h.) in inches of the tree(s) torching out.	Press [A].
--Enter height in feet of tree(s) torching out.	Press [B].
--Enter mean cover height, in feet, of the area downwind of the firebrand source. Albini called this "mean treetop height and uses this value to characterize the general forest cover as it influences the wind. If there is broken forest cover, he suggests using half the treetop height of the forest-covered portion. If there is little or no forest cover, enter vegetation height.	Press [C].
--Enter average windspeed, in miles per hour, 20 ft above the vegetation.	Press [D].
--Enter the number of identical trees burning at once to produce a single flame.	Press [E].
--Enter elevational difference in feet from ridgetop to valley bottom as would be shown on a map. Note that the entry is in feet even though the equations use multiples of 1,000 ft. The calculator makes the required conversion. The assumption made in Albini's model is that terrain resembles a washboard. If this simple representation of terrain does not describe your situation, perhaps the model will not give you a good approximation of spotting distance.	Press [2nd] [A].
--Enter the ridgetop-to-valley-bottom horizontal distance in miles as would be shown on a map.	Press [2nd] [B].

Name ____________________ Date ____________ Sheet ____ of ____

Purpose ____

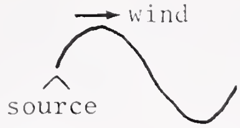
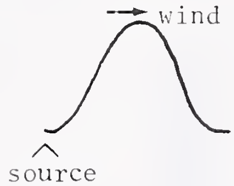
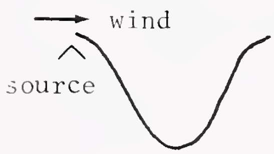
INPUT	KEY					Reg. No.
*Species	SBR STO					
*Torching tree d.b.h., inch	A					34
*Torching tree height, ft	B					35
Mean cover height, ft	C					36
20-foot windspeed, mi/h	D					37
*Number of trees torching together	E					38
Ridge/valley elevational difference, ft	2nd A					39
Ridge/valley horizontal distance, mi	2nd B					40
Spotting source location code 0 - midslope, windward side 1 - valley bottom 2 - midslope, leeward side 3 - ridgetop	2nd C					41
**Continuous flame height, pile burning, ft	2nd E					42
OUTPUT						
Flat-terrain max. spot. distance, mi	SBR =					22
Mountainous-terrain max, spot. distance, mi	SBR 2nd =					23
Flame height, ft	R/S					
Torching tree option (flashing)						24
or						
Pile burning option (continuous)						42

*Not needed for the pile burning option

**An entry specifies pile burning option; no entry specifies torching tree option

Figure 1.--TI-59 MAXIMUM SPOTTING DISTANCE WORKSHEET

--Enter the spotting source location code from the following list: Press $\boxed{\text{2nd}}$ $\boxed{\text{C}}$.

Enter	for	
0	midslope, windward side	
1	valley bottom	
2	midslope, leeward side	
3	ridgetop	

--OMIT FOR TORCHING TREE OPTION: Enter estimated flame height in feet from observation of continuous flame (see fig. 2). This value signals the calculator to use the pile burning option and must be entered for each run of this option even if it remains unchanged. (Entry of 0 here while using the torching tree option will cause a flashing display of an erroneous flat-terrain spotting distance in the output section. If this occurs, press $\boxed{\text{CLR}}$ and start the run again.) Press $\boxed{\text{2nd}}$ $\boxed{\text{E}}$.

III. Recall and correction of input

1. Recall first input item by pressing $\boxed{\text{SBR}}$ $\boxed{\text{RCL}}$. Follow with a series of $\boxed{\text{R/S}}$ to obtain successive inputs in the order listed on the worksheet. The recall subroutine can be entered only at the beginning of the list.

   For mean cover height, the value recalled will depend on whether $\boxed{\text{SBR}}$ $\boxed{\text{RCL}}$ is performed before or after calculations. Before calculations,

the mean cover height that was input will be displayed. After calculations, $\overline{h}^*$ will be displayed (see equation summary).

When continuous flame height is recalled, a steady numerical display of that value indicates that the pile burning option is being run. Flashing nines at that point indicate that the torching tree option is being used, so no continuous flame height should exist. Press [CLR] and continue.

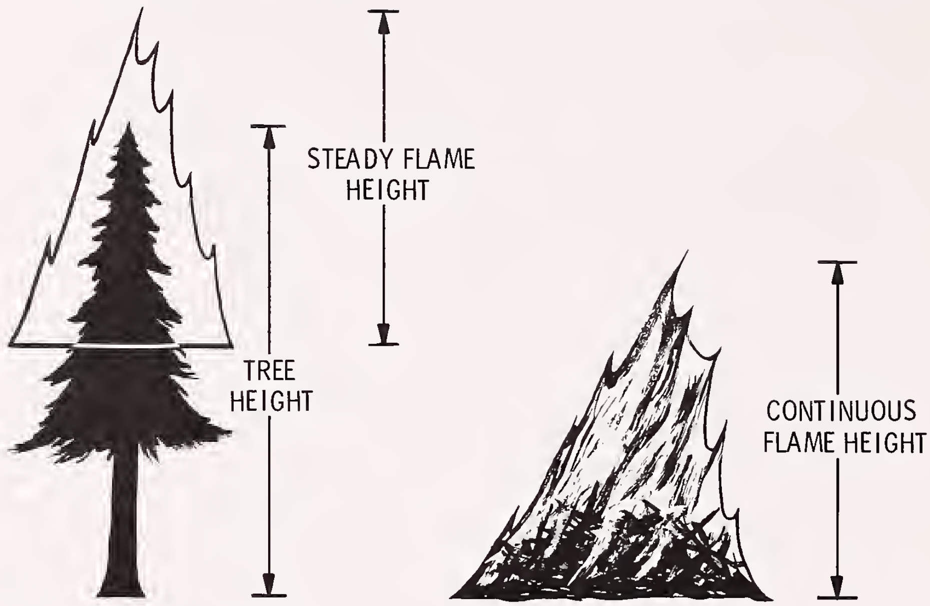


Figure 2.--The steady flame height is the perpendicular distance from the base of the flame (within the crown of the tree) to the tip of the flame. Continuous flame height is the distance from the ground to the tip of the flame.

2. To change any item(s) of input, re-enter the desired value as in part II, step 3.

IV. Performing calculations

1. Press [SBR] [=]. When calculations cease, the flat-terrain spotting distance in miles will appear in the display. Record it on the worksheet.

2. Press [SBR] [2nd] [=]. The flat-terrain spotting distance is corrected for mountainous terrain, and the map location spotting distance in miles is displayed. *Step 1 must be performed prior to this step for each run.* Record the result on the worksheet.

3. Press [R/S]. *This step must follow immediately after step 2.* The flame height (ft) will be displayed. In the pile burning option, the display of the input value will be continuous. In the torching tree option, the calculated flame height will be a flashing display. Note that this is not the distance from the ground to the tip of the flame (see fig. 2). After recording the value, press [CLR] to halt the flashing display.

V. Making successive runs

1. Change species data, if necessary.

2. Repeat part II, step 2 (SBR CLR).

3. Change any input values desired.

4. Enter estimated flame height if the pile burning option is being run.

5. Perform calculations.

WORKED EXAMPLES

Figure 3 contains the inputs and outputs of two sample problems--one for the torching tree option and one for the pile burning option.

CONDENSED INSTRUCTIONS

I. Press 1, feed side 1 (flashing: CLR , try again). Press 2, feed side 2 (flashing: CLR , try again).

II. 1. SBR STO . Enter species data card.

2. SBR CLR .

3.

Input	Press
D.b.h. (inch)	A
Height (ft)	B
Mean cover height (ft)	C
Wind (mi/h)	D
Number of trees	E
Ridge/valley elevation difference (ft)	2nd A
Ridge/valley horizontal distance (mi)	2nd B
Spotting source location code	2nd C
0 - midslope, windward side	
1 - valley bottom	
2 - midslope, leeward side	
3 - ridgetop	
Continuous flame height (ft)	2nd E

III. SBR RCL ; follow by series of R/S

2nd R/S mean cover height: { value input displayed before calculations; $\bar{h}$* displayed after calculations }

8th R/S continuous flame height: { value input displayed in pile burning option; flashing nines in torching tree option }

IV. SBR = : display flat-terrain spotting distance (mi)

SBR 2nd = : display mountainous-terrain spotting distance (map mi)

R/S : display flame height (ft)--flashing display in torching tree option, continuous display in pile burning option

Figure 3.--TI-59 MAXIMUM SPOTTING DISTANCE WORKSHEET. Sample problems.

Name ______________ Date ______________ Sheet ____ of ____

Purpose ______________

INPUT	KEY					Reg. No.
*Species	SBR STO	Grand fir	----			
*Torching tree d.b.h., inch	A	20	----			34
*Torching tree height, ft	B	137	----			35
Mean cover height, ft	C	130	100			36
20-foot windspeed, mi/h	D	20	15			37
*Number of trees torching together	E	1	----			38
Ridge/valley elevational difference, ft	2nd A	4000	2000			39
Ridge/valley horizontal distance, mi	2nd B	.25	1			40
Spotting source location code 0 - midslope, windward side 1 - valley bottom 2 - midslope, leeward side 3 - ridgetop	2nd C	3	1			41
**Continuous flame height, pile burning, ft	2nd E	----	45			42
OUTPUT						
Flat-terrain max. spot. distance, mi	SBR =	0.34	0.25			22
Mountainous-terrain max, spot. distance, mi	SBR 2nd =	0.31	0.21			23
Flame height, ft	R/S					
Torching tree option (flashing)		77				24
or						
Pile burning option (continuous)			45			42

*Not needed for the pile burning option
**An entry specifies pile burning option; no entry specifies torching tree option

REGISTER ASSIGNMENTS

Register	Symbol	Contents
34	d	Torching tree d.b.h.
35	h	Torching tree height
36	$\left\{\begin{array}{l}\overline{h}_* \\ \overline{h}\end{array}\right.$	Mean cover height input Mean cover height used in computation
37	U	20-foot windspeed
38	n	Number of trees torching together
39	1000H	Ridge/valley elevation difference
40	D	Ridge/valley horizontal distance
41	M	Spotting source location code
42	H_F	Continuous flame height (pile burning)
24	h_F	Adjusted steady flame height
25	d_F	Adjusted flame duration
28	z(0)	Initial firebrand height
22	F	Flat-terrain spotting distance
23	S	Mountainous-terrain spotting distance

PUBLICATIONS CITED

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1981. Spot fire distance from isolated sources--extensions of a predictive model. USDA For. Serv. Res. Note INT-309, 9 p. Intermt. For. and Range Exp. Stn., Ogden, Utah.

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1979. Spot-fire distance from burning trees--a predictive model. USDA For. Serv. Res. Pap. INT-56, 73 p. Intermt. For. and Range Exp. Stn., Ogden, Utah.

APPENDIX

FORMULATION OF EQUATIONS

Equations were generated that approximate the curves for flame height and duration and for firebrand height in the FBO field reference. They were obtained by regressing points from the FBO curves to get power curve approximations. These equations appear in the figures accompanying the step-wise summary of the solution method presented below. The steps formed the basis of the FBO method of manually determining spotting distance and provide the rationale behind the equations presented.

1. The steady flame height is determined by relationships developed by Albini (1979) through the artifice of relationships between tree diameter and foliar biomass. The amount of foliar biomass and how it burns provides the linkage between diameter and flame height. Curves fitted to these relationships are shown in figure 4. Then, to obtain the equations appearing in the summary, the steady flame height is corrected by a factor of $n^{0.4}$ (Albini 1979), where n is the number of trees burning simultaneously.

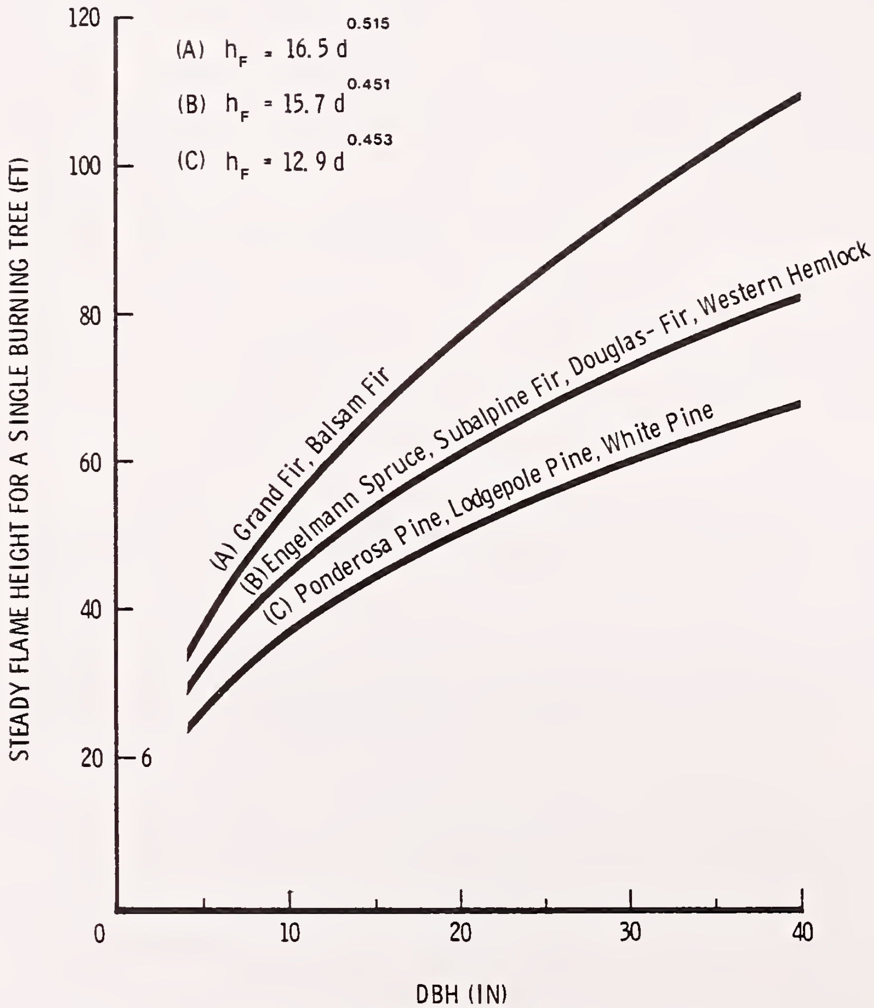

Figure 4.--Height of "steady" flame from burning of one tree crown in still air.

2. Dimensionless steady flame duration is determined by relationships developed by Albini (1979), again through the artifice of relationships between tree diameter and foliar biomass. Curves fitted to these relationships are shown in figure 5. Then, to obtain the equations appearing in the summary, the steady flame duration is corrected by a factor of $n^{-0.2}$ (Albini 1979), where n is the number of trees burning simultaneously.

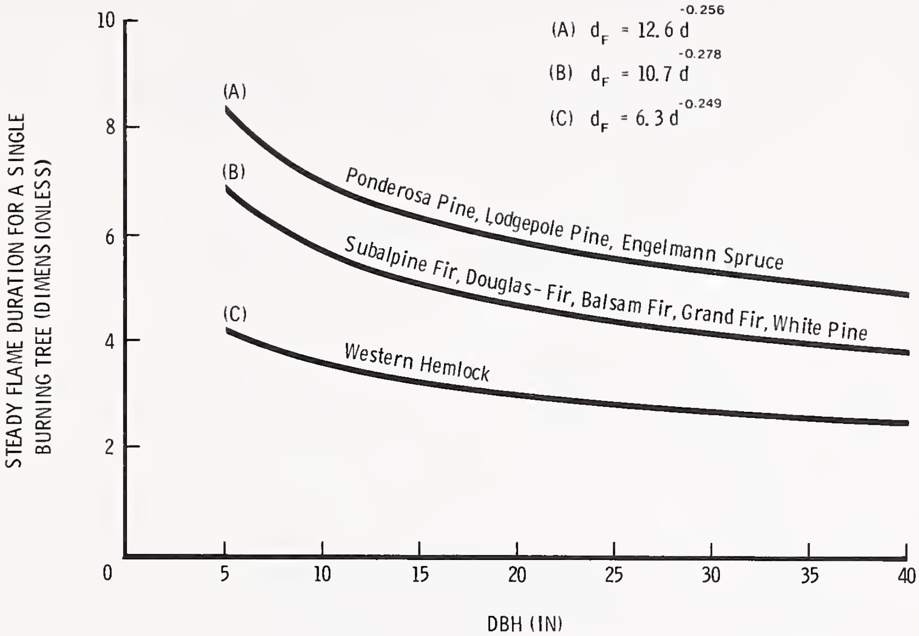

Figure 5.--Steady flame duration for individual tree torching out.

3. The ratio of lofted firebrand height to steady flame height is determined using figure 6. That quantity is multiplied by steady flame height and added to half of the torching tree's height to obtain initial firebrand height, z(0).

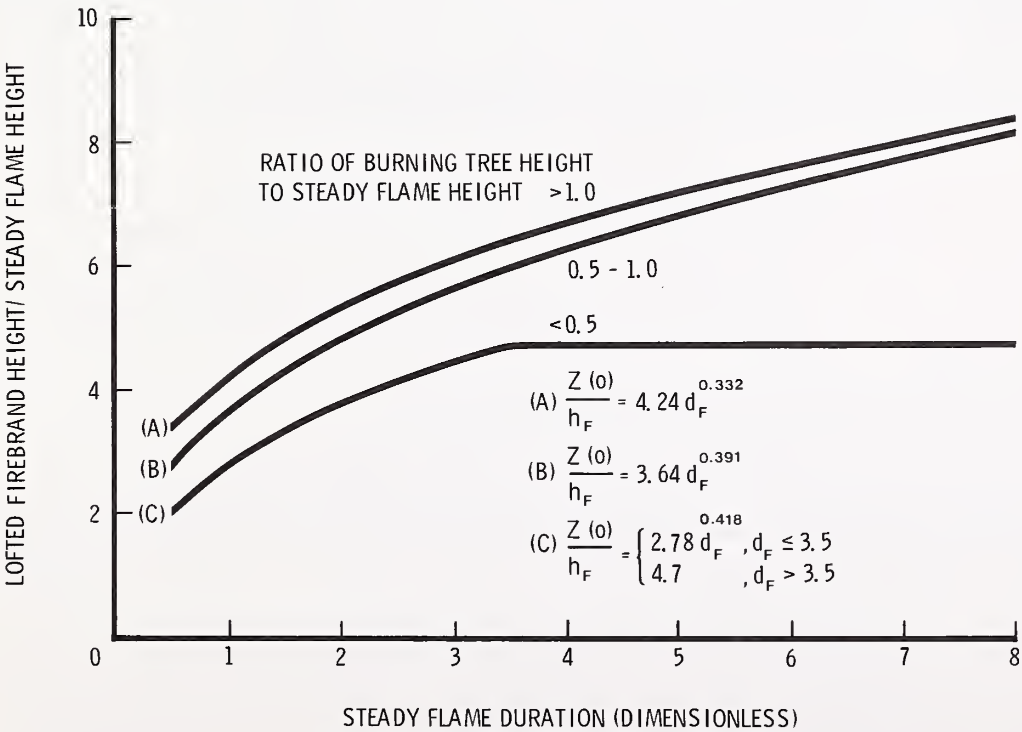

Figure 6.--Height of firebrands lofted by burning trees.

When the firebrand-lofting flame is continuous as opposed to the brief transitory flame from a torching tree, the first three steps above are replaced by a simple multiplication. The maximum height of a viable firebrand lofted by a continuous flame of height H_F is given by $z(0) = 12.2H_F$ (Albini 1981).

4. Flat-terrain spotting distance is calculated next. Albini's equation for flat-terrain spotting distance over forest-covered terrain is:

$$F = 21.9U_h\left(\frac{\bar{h}}{g}\right)^{1/2}\left\{0.362 + \left(\frac{z(0)}{\bar{h}}\right)^{1/2}\frac{1}{2}\ln\left(\frac{z(0)}{\bar{h}}\right)\right\}$$

where

F = flat-terrain spotting distance (mi)

U_h = wind at treetop height (mi/h)

$\bar{h}$ = mean height of vegetation cover along the firebrand's flight path (ft)

g = acceleration of gravity (ft/h^2)

$z(0)$ = initial firebrand height (ft).

Using the assumption that U_h is two-thirds of the 20-ft wind, substituting $g = 32$ ft/s^2 and converting seconds to hours, the equation becomes

$$F = 7.18 \times 10^{-4}\ U\ \bar{h}^{1/2}\left\{0.362 + \left(\frac{z(0)}{\bar{h}}\right)^{1/2}\frac{1}{2}\ln\left(\frac{z(0)}{\bar{h}}\right)\right\},$$

where U = windspeed at 20 ft (mi/h). If transport is over terrain which is not forest-covered, then the "effective" height, $\bar{h}_c$, is used in the above equation instead of $\bar{h}$ (Albini 1981).

5. The flat-terrain spotting distance must be corrected for use in mountainous terrain. Albini's equation for maximum horizontal distance is

$$F = S + \frac{(1 + \alpha)a}{m(z_G - \bar{z}_T)}\left\{\cos\left(m(S + X_1)\right) - \cos(mX_1)\right\}$$

where

F = flat-terrain spotting distance (mi)

S = mountainous-terrain spotting distance (mi)

$2a$ = total elevation difference, ridge to valley = ΔH_{RV} (ft)

D = horizontal distance from ridge to valley (map mi)

$m = \pi/D$

X_1 = distance of firebrand source from windward midslope (map mi)

$\bar{z}_T$ = mean terrain height above sea level (ft)

z_G = altitude of top of surface-influenced airflow layer (ft)

α = a dimensionless constant related to the gross structure of the surface-influenced atmosphere.

Using $\alpha = 0.2$ and $(z_G - \bar{z}_T) = 6{,}000$ ft, we have

$$F = S + \frac{(1.2)\Delta H_{RV}/2}{6{,}000\pi/D}\left\{\cos(\pi \frac{S}{D} + mX_1) - \cos(mX_1)\right\}.$$

So

$$\frac{F}{D} = \frac{S}{D} + \frac{\Delta H_{RV}}{\pi(10^4)}\left\{\cos(\pi \frac{S}{D} + mX_1) - \cos(mX_1)\right\}.$$

Let $H = \Delta H_{RV}/1000$, $F/D = A$, and $S/D = X$. Thus

$$X = A - \frac{H}{10\pi}\left\{\cos(\pi X + mX_1) - \cos(mX_1)\right\}.$$

Consider the four cases:

mX_1	Location of firebrand source
0	midslope on windward side of ridge
$-\pi/2$	valley bottom
$-\pi$	midslope on leeward side of ridge
$-3\pi/2$	ridgetop

So $mX_1 = -M\pi/2$, where M is 0, 1, 2, or 3, respectively, for the cases listed. Let $B = H/10\pi$, then

$$X = A - B\left\{\cos(\pi X - M\pi/2) - \cos(M\pi/2)\right\}.$$

This equation can be solved by the iteration:

$$X_o = A$$

$$X_{n+1} = A - B\left\{\cos(\pi X_n - M\pi/2) - \cos(M\pi/2)\right\}.$$

X_n converges sufficiently within six iterations for use here.

SPECIES DATA CARDS

To make a species data card, the formulas for steady flame height and for flame duration must be expressed as power curves of the form (see, for example, the curves in Albini 1979):

$$h_F = xd^y$$

$$d_F = zd^w$$

where

h_F = steady flame height, ft

d_F = flame duration (dimensionless)

d = diameter at breast height, inch.

Using the equation for steady flame height and taking the logarithm of both sides

$$\ln h_F = \ln x + y \ln d.$$

When several points (d, h_F) are chosen on the curve, linear regression can be used to fit a straight line to the data points[3] $(\ln d, \ln h_F)$. The y-intercept will be ln x and the slope will be y.

The derivation of the formula for flame duration parallels that for steady flame height.

Then, the list of data register contents for the data card is:

Register number	Contents	
00	0	
01	x	
02	y	
03	z	
04	w	
05	4.24	Constants applicable to all species
06	0.332	
07	3.64	
08	0.391	
09	2.78	
10	0.418	
11	4.7	
12-29	0	

To prepare the data card:

1. Turn on your calculator. If it is already on, turn it off momentarily to clear all storage registers.

2. For *each* register numbered 1 through 11: Enter the contents from the list into the display and press $\boxed{\text{STO}}$ nn where nn is the register number.

---

[3]The TI-59 applied statistics module contains a curve-fitting program (ST-12) that accepts as input the points (d, h_F) and provides x and y as output. This eliminates the transformation to $(\ln d, \ln h_F)$.

3. Press $\boxed{4}$ $\boxed{2nd}$ $\boxed{R/S}$ and feed in one side of the blank data card. A 4. should appear in the display indicating that the data has been recorded. If the display flashes, press $\boxed{CLR}$ and repeat step 3.

4. Repeat step 3, feeding in the other side of the data card.

PROGRAM DUPLICATION

Program Cards

1. Turn on your calculator. Enter the program into memory by performing part I of the operating procedure using the program card to be duplicated.

2. Press $\boxed{1}$ $\boxed{2nd}$ $\boxed{R/S}$ and feed in side 1 of the blank program card. If the display flashes, press $\boxed{CLR}$ and repeat step 2.

3. Press $\boxed{2}$ $\boxed{2nd}$ $\boxed{R/S}$ and feed in side 2 of the blank program card. If the display flashes, press $\boxed{CLR}$ and repeat step 3.

Data Cards

1. Press $\boxed{4}$ and feed in one side of the data card to be duplicated. If the display flashes, press $\boxed{CLR}$ and try again.

2. Press $\boxed{4}$ $\boxed{2nd}$ $\boxed{R/S}$ and feed in one side of the blank card. If the display flashes, try again. Repeat step 2 for the other side of the blank card.

PROGRAM LISTING

000	22	INV
001	58	FIX
002	22	INV
003	86	STF
004	01	01
005	00	0
006	42	STO
007	42	42
008	91	R/S
009	76	LBL
010	42	STO
011	22	INV
012	58	FIX
013	04	4
014	92	RTN
015	76	LBL
016	11	A
017	42	STO
018	34	34
019	92	RTN
020	76	LBL
021	12	B
022	42	STO
023	35	35
024	92	RTN
025	76	LBL
026	13	C
027	42	STO
028	36	36
029	92	RTN
030	76	LBL
031	14	D
032	42	STO
033	37	37
034	92	RTN
035	76	LBL
036	15	E
037	42	STO
038	38	38
039	92	RTN
040	76	LBL
041	16	A'
042	42	STO
043	39	39
044	92	RTN
045	76	LBL
046	17	B'
047	42	STO
048	40	40
049	92	RTN
050	76	LBL
051	18	C'
052	42	STO
053	41	41
054	92	RTN
055	76	LBL
056	10	E'
057	42	STO
058	42	42
059	86	STF
060	01	01
061	92	RTN
062	76	LBL
063	95	=
064	70	RAD
065	87	IFF
066	01	01
067	32	X:T
068	71	SBR
069	22	INV
070	71	SBR
071	23	LNX
072	76	LBL
073	52	EE
074	43	RCL
075	31	31
076	32	X:T
077	01	1
078	22	INV
079	77	GE
080	45	Y^X
081	93	.
082	05	5
083	22	INV
084	77	GE
085	34	√X
086	43	RCL
087	25	25
088	32	X:T
089	03	3
090	93	.
091	05	5
092	77	GE
093	35	1/X
094	01	1
095	01	1
096	42	STO
097	26	26
098	01	1
099	02	2
100	42	STO
101	27	27
102	61	GTO
103	53	(
104	76	LBL
105	35	1/X
106	09	9
107	42	STO
108	26	26
109	01	1
110	00	0
111	42	STO
112	27	27
113	61	GTO
114	53	(
115	76	LBL
116	34	√X
117	07	7
118	42	STO
119	26	26
120	08	8
121	42	STO
122	27	27
123	61	GTO
124	53	(
125	76	LBL
126	45	Y^X
127	05	5
128	42	STO
129	26	26
130	06	6
131	42	STO
132	27	27
133	76	LBL
134	53	(
135	71	SBR
136	24	CE
137	76	LBL
138	54	)
139	71	SBR
140	44	SUM
141	58	FIX
142	02	02
143	91	R/S
144	76	LBL
145	90	LST
146	00	0
147	32	X:T
148	43	RCL
149	40	40
150	67	EQ
151	85	+
152	53	(
153	43	RCL
154	22	22
155	55	÷
156	43	RCL
157	40	40
158	54	)
159	42	STO
160	32	32
161	42	STO
162	30	30
163	53	(
164	43	RCL
165	39	39
166	55	÷
167	01	1
168	00	0
169	00	0
170	00	0
171	00	0
172	55	÷
173	89	π
174	54	)
175	42	STO
176	33	33
177	06	6
178	42	STO
179	00	00
180	76	LBL
181	71	SBR
182	53	(
183	43	RCL
184	32	32
185	75	−
186	43	RCL
187	33	33
188	65	×
189	53	(
190	53	(
191	89	π
192	65	×
193	43	RCL
194	30	30
195	75	−
196	43	RCL
197	41	41
198	65	×
199	89	π
200	55	÷
201	02	2
202	54	)
203	39	COS
204	75	−
205	53	(
206	43	RCL
207	41	41
208	65	×
209	89	π
210	55	÷
211	02	2
212	54	)
213	39	COS
214	54	)
215	54	)
216	42	STO
217	30	30
218	97	DSZ
219	00	00
220	71	SBR
221	53	(
222	43	RCL
223	30	30
224	65	×
225	43	RCL
226	40	40
227	54	)
228	42	STO
229	23	23
230	91	R/S
231	76	LBL
232	55	÷
233	87	IFF
234	01	01
235	65	×

```
236  43  RCL
237  24   24
238  33  X^2
239  94  +/-
240  34  √X
241  91  R/S
242  76  LBL
243  65   ×
244  43  RCL
245  42   42
246  91  R/S
247  76  LBL
248  43  RCL
249  43  RCL
250  34   34
251  91  R/S
252  43  RCL
253  35   35
254  91  R/S
255  43  RCL
256  36   36
257  91  R/S
258  43  RCL
259  37   37
260  91  R/S
261  43  RCL
262  38   38
263  91  R/S
264  43  RCL
265  39   39
266  91  R/S
267  43  RCL
268  40   40
269  91  R/S
270  43  RCL
271  41   41
272  91  R/S
273  22  INV
274  87  IFF
275  01   01
276  33  X^2
277  43  RCL
278  42   42
279  92  RTN
280  76  LBL
281  33  X^2
282  00   0
283  35  1/X
284  92  RTN
285  76  LBL
286  22  INV
287  53   (
288  43  RCL
289  01   01
290  65   ×
291  43  RCL
292  34   34
293  45  Y^X
294  43  RCL
295  02   02
296  65   ×
297  43  RCL
298  38   38
299  45  Y^X
300  93   .
301  04   4
302  00   0
303  05   5
304  54   )
305  53   (
306  42  STO
307  24   24
308  35  1/X
309  65   ×
310  43  RCL
311  35   35
312  54   )
313  42  STO
314  31   31
315  92  RTN
316  76  LBL
317  23  LNX
318  53   (
319  43  RCL
320  03   03
321  65   ×
322  43  RCL
323  34   34
324  45  Y^X
325  43  RCL
326  04   04
327  65   ×
328  43  RCL
329  38   38
330  45  Y^X
331  93   .
332  01   1
333  09   9
334  09   9
335  94  +/-
336  54   )
337  42  STO
338  25   25
339  92  RTN
340  76  LBL
341  24  CE
342  53   (
343  73  RC*
344  26   26
345  65   ×
346  43  RCL
347  25   25
348  45  Y^X
349  73  RC*
350  27   27
351  65   ×
352  43  RCL
353  24   24
354  85   +
355  43  RCL
356  35   35
357  55   ÷
358  02   2
359  54   )
360  53   (
361  42  STO
362  28   28
363  45  Y^X
364  93   .
365  03   3
366  03   3
367  07   7
368  65   ×
369  02   2
370  93   .
371  02   2
372  75   -
373  04   4
374  93   .
375  00   0
376  54   )
377  32  X:T
378  43  RCL
379  36   36
380  22  INV
381  77   GE
382  28  LOG
383  92  RTN
384  76  LBL
385  28  LOG
386  32  X:T
387  42  STO
388  36   36
389  92  RTN
390  76  LBL
391  32  X:T
392  53   (
393  43  RCL
394  42   42
395  65   ×
396  01   1
397  02   2
398  93   .
399  02   2
400  54   )
401  53   (
402  42  STO
403  28   28
404  55   ÷
405  43  RCL
406  36   36
407  54   )
408  42  STO
409  29   29
410  61  GTO
411  54   )
412  76  LBL
413  44  SUM
414  53   (
415  43  RCL
416  28   28
417  55   ÷
418  43  RCL
419  36   36
420  54   )
421  53   (
422  42  STO
423  29   29
424  23  LNX
425  55   ÷
426  02   2
427  65   ×
428  43  RCL
429  29   29
430  34  √X
431  85   +
432  93   .
433  03   3
434  06   6
435  02   2
436  54   )
437  65   ×
438  53   (
439  43  RCL
440  36   36
441  55   ÷
442  03   3
443  02   2
444  54   )
445  34  √X
446  65   ×
447  43  RCL
448  37   37
449  65   ×
450  93   .
451  00   0
452  00   0
453  04   4
454  00   0
455  06   6
456  54   )
457  42  STO
458  22   22
459  92  RTN
460  76  LBL
461  25  CLR
462  81  RST
463  76  LBL
464  85   +
465  43  RCL
466  22   22
467  42  STO
468  23   23
469  91  R/S
470  71  SBR
471  55   ÷
472  00   0
473  00   0
474  00   0
475  00   0
476  00   0
477  00   0
478  00   0
479  00   0
```

The Intermountain Station, headquartered in Ogden, Utah, is one of eight regional experiment stations charged with providing scientific knowledge to help resource managers meet human needs and protect forest and range ecosystems.

The Intermountain Station includes the States of Montana, Idaho, Utah, Nevada, and western Wyoming. About 231 million acres, or 85 percent, of the land area in the Station territory are classified as forest and rangeland. These lands include grasslands, deserts, shrublands, alpine areas, and well-stocked forests. They supply fiber for forest industries; minerals for energy and industrial development; and water for domestic and industrial consumption. They also provide recreation opportunities for millions of visitors each year.

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